ACT460H1F/STA2502 Fall 2019
Stochastic Methods for Actuarial Science

Instructor: X. Sheldon Lin Office: 402 Stewart Building, 149 College St.
Email: sheldon@utstat.utoronto.ca
Website: `www.utstat.utoronto.ca/sheldon/teaching.html`
Office hours: Thursdays 1pm-3pm or by appointment.
Lecture times and location: Tuesdays 2:10-5pm, GB248.

Prerequisites: STA347
ACT370 is strongly recommended. If you don't have it. you must have a strong background in probability and some knowledge in finance/financial economics in order to have a reasonable chance to pass the course. If you are a transfer student or for some reason you did not take STA347 but want to take this course, according the FAS regulations, you must submit a waiver form
(`http://www.utstat.toronto.edu/wordpress/wp-content/uploads/2011/09/request-for-prereq-or-coreq-waiver.pdf`)
to me in person for approval. Please submit a filled waiver form by Tuesday Sept 17, or you will be removed from the course on Wednesday Sept 19.

Recommended Reference Books
I will provide you detailed lecture notes for this course but do encourage you to purchase **Financial Calculus: An Introduction to Derivative Pricing** by Martin Baxter and Andrew Rennie, Cambridge University Press. It should be available from amazon.ca.
The following books are helpful.
Introductory Stochastic Analysis for Finance and Insurance by X. Sheldon Lin, Wiley and Sons. It provides advanced mathematical modelling tools for option pricing.
Derivatives Markets by Robert MacDonald. This is a book for MBAs in finance and hencec not mathematical. But it describes financial markets in 'layman's' terms.
Options, Futures and Other Derivatives by John Hull, Prentice Hall. It is the bible for option pricing with an accessible math level.

Learning Objectives/Outcomes

A. General Properties of Options
Students will be able to:
1. Define and recognize the definitions of call and put options, expiration date, strike price/exercise price, moneyness, European option, American option, payoff and net profit of long and short option positions;
2. Apply put-call parity to European options on the following underlying assets:
Stock (no dividends, discrete and continuous dividends), currency, futures contract;

B. The Binomial Option Pricing Model Students will be able to:
1. Apply the principle of no-arbitrage and identify arbitrage opportunities if any;

2. Apply the risk-neutral pricing formula for pricing European and American options;
3. Construct a binomial model from market stock price data using historical volatility.

C. The Black-Scholes Option Pricing Model
Students will be able to:
1. Recognize the underlying assumptions behind the Black-Scholes model;
2. Explain the properties of a lognormal distribution and calculate the following for future stock prices under the Black-Scholes model;
3. Derive the analytic pricing formulas for the following European options using risk-neutral pricing formulas; 4. Implement the risk-neutral pricing formula using Monte-Carlo simulation.

D. Option Greeks and Risk Management
Students will be able to:
1. Interpret and compute option Greeks (Delta, Gamma, Theta, Vega, Rho, and Psi);
2. Approximate option prices using delta, gamma and theta.
3. Explain and demonstrate how to control stock price risk using the methods of delta-hedging and gamma-hedging.

E. Interest Rate, Bonds, and Interest Rate Derivatives
Students will be able to:
Price bonds and interest rate derivatives under the Vasicek and CIR interest rate models.

Topics

I will use **Financial Calculus** as a main reference and cover the topics in the following order. However, the materials to be covered in the course will be substantially more than those in the book in terms of breadth and depth.

- Mathematical Foundations
  - the binomial distribution
  - the normal and lognormal distributions
  - expectation, volatility and Laplace transform
  - the central limit theorem and the law of large numbers
  - time value of money, force of interest, money market account
  - stock models: binomial model and geometric Brownian motion
  - actuarial pricing vs no-arbitrage pricing
  - perfect hedging: forward contract as an example
- Discrete Processes and Binomial Trees
  - one-period binomial model/tree

  - no-arbitrage pricing and risk-neutral probabilities
  - random walk and binomial stock model
  - multi-period recombining binomial tree and the distribution of the risky asset at any given time
  - risk-neutral probability measure
  - pricing options by backwards recursion
  - the CRR formula, pricing options using the binomial distribution
  - Excel and VBA programming for option pricing
  - American option and valuation
  - self-financing strategy, replicating portfolio
  - complete and incomplete markets
  - construction of binomial models from market information
  - valuation and hedging of variable annuity guarantees with binomial model
  - moving towards continuous models

- Continuous Processes
  - (arithmetic) Brownian motion and properties
  - geometric Brownian motion model for stock prices
  - stochastic integral and properties
  - Ito process and SDEs
  - SDE expression of BM and GBM
  - Ito's lemma and its applications
  - the product rule
  - martingale revisit
  - present value process and martingale
  - Fundamental Theorem of Asset Pricing
  - risk-neutral valuation
  - self-financing trading strategy and replicating portfolio in continuous time
  - application to the Black-Scholes model
  - The Black-Scholes PDE
  - greeks (Delta, Gamma, Rho and Vega) and sensitivity analysis
  - implied volatility
  - simulating stock price paths under the real-world and risk-neutral probability measures, and pricing option by simulation, using Excel and VBA
  - dynamic hedging variable annuities using greeks

- Pricing Market and Fixed Income Securities

– currency exchange and exchange forwards and options
– options on stocks with dividends
– zero-coupon/discount bonds and coupon bonds
– zero rate, bond yield, short rate and forward rate, and their term structures
– SDE for zero-coupon bonds, the Sharp ratio
– risk-neutral valuation for bonds
– the extended Vasicek/Hull-White short rate model and its properties
– the affine form of zero-coupon bond prices under Vasicek
– calibration of the extended Vasicek model to zero curve
– simulating short rate paths
– the Cox, Ingersoll and Ross model and its applications
– derivatives on discount and coupon-bearing bonds

Assignments, Test and Exam
I will give 5 homework assignments during the semester. One half of the problems in each assignment will be graded. Each assignment is worth 3% toward to the final mark. There will be 2 tutorials conducted by a TA to cover EXCEL and VBA programming skills for financial modelling. A 90-minutes written-answer midterm test will take place on Tuesday Oct 22 from 2:15 to 3:45pm. Location: TBA. It will account for 34% of the final mark. Should you be forced to miss the test, you are required by faculty regulations to submit, within one week, appropriate documentation from the U of T Health Services to me or to the Departmental Office SS6018 (Print on it your NAME, STUDENT NUMBER, course number, and date.). **And you must contact me to arrange a time within one week for an individual oral makeup test.** A written-answer final exam (2 hours) will be given during the faculty exam period. The final exam accounts for 51% of the final mark.

The Code of Behaviour on Academic Matters
Visit `www.artsci.utoronto.ca/osai/students`

Relation with the SOA MFE Exam
This course is not designed to prepare for the SOA MFE exam. The topics and mathematical contents covered in this course go much deeper and broader than those in the MFE, especially after the recent changes in the MFE syllabus. However, the course does help you understand the topics in the exam.

Canadian Institute of Actuaries (CIA)'s University Accreditation Program (UAP)

ACT460 is an accredited course under the UAP program. The minimum grade needed to apply for an exemption is 75. For detailed information on UAP, please visit the following webpages:

- University Accreditation Program description (`http://www.cia-ica.ca/membership/uap`)
- List of accredited courses offered by University of Toronto:
  `http://www.cia-ica.ca/membership/uap/accredited/toronto`
- How to apply for CIA exemptions:
  `http://www.cia-ica.ca/membership/uap/information-for-students`

Note: The CIA will grant credits to students for SOA/CAS examinations based on the achievement of the minimum Grade towards Associateship (ACIA) and Felllowship (FCIA) in the CIA. At the time of this agreement, CIA credits are recognized by the following actuarial organizations towards their respective designations:
Casualty Actuarial Society (CAS): ACAS, FCAS
UK Institute and Faculty of Actuaries (IFoA): FIA, AIA
Institute of Actuaries of Australia (IAA): AIAA, FIAA
Actuarial Society of South Africa (ASSA): AMASSA, FASSA
American Academy of Actuaries (AAA): MAAA

The CIA does not guarantee that credits granted to students under the CIA UAP will be recognized by any other actuarial organizations towards their actuarial designations.