ACT460H1F/STA2502 Fall 2022
Stochastic Methods for Actuarial Science

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Office hours: Thursdays from 1-3pm in person. If you can not make it make an appointment with me.
Lectures: Tuesdays 2:10pm-5pm; Location: Medical Science Building MS2173.
Zoom link in case we need to switch the lectures online:
`https://utoronto.zoom.us/j/87965030345`
Please note that there are no passwords nor waiting room for these zoom meetings so you can join in any time.

Prerequisites: ACT350 or STA347
ACT370 is strongly recommended. If you don't have it. you must have a very strong background in probability and some knowledge in finance/financial economics in order to have a reasonable chance to pass the course. If you are a transfer student or for some reason you did not take STA347 but want to take this course, please contact me within two weeks. If you fail to do so, Prof Zhang will remove you from the class at the end of two weeks.

Recommended Reference Books
I will provide you detailed notes for this course but do encourage you to purchase
Financial Calculus: An Introduction to Derivative Pricing by Martin Baxter and Andrew Rennie, Cambridge University Press. This book provides a good overview of the topics. It is available from amazon.ca.
The following books are also helpful.
Introductory Stochastic Analysis for Finance and Insurance by X. Sheldon Lin, Wiley and Sons. If you wish to better understand the mathematics and probability theory underlying the topics in this course.
Derivatives Markets by Robert MacDonald. A book for MBAs so it focuses on the aspects of financial markets.
Options, Futures and Other Derivatives by John Hull, Prentice Hall. The very popular book for financial quants.

Topics
I will use Financial Calculus as a main reference and cover the topics in the following order. However, the materials to be covered in the course will be substantially more than those in the book in terms of breadth and depth.

Learning Objectives/Outcomes

A. General Properties of Options
Students will be able to:
1. Define and recognize the definitions of call and put options, expiration date, strike price/exercise price, moneyness, European option, American option, payoff and net profit of long and short option positions;
2. Apply put-call parity to European options on the following underlying assets:
Stock (no dividends, discrete and continuous dividends), currency, futures contract;
B. The Binomial Option Pricing Model Students will be able to:
1. apply the principle of no-arbitrage, and identify arbitrage opportunities if any;
2. applying the risk-neutral pricing formula for pricing European and American options;
3. Construct a binomial model from market stock price data using historical volatility.
C. The Black-Scholes Option Pricing Model
1. Recognize the underlying assumptions behind the Black-Scholes model;
2. Explain the properties of a lognormal distribution and calculate the following for future stock prices under the Black-Scholes model;
3. Deduce the analytic pricing formulas for the following European options using risk-neutral pricing formulas; 4. Implement the risk-neutral pricing formula using Monte-Carlo simulation:
D. Option Greeks and Risk Management
Students will be able to:
1. Interpret and compute option Greeks (Delta, Gamma, Theta, Vega, Rho, and Psi);
2. Approximate option prices using delta, gamma and theta.
3. Explain and demonstrate how to control stock price risk using the methods of delta-hedging and gamma-hedging.
E. Interest Rate, Bonds, and Interest Rate Derivatives
Students will be able to:
Price bonds and interest rate derivatives under the Vasicek and CIR interest rate models.

Topics

- Mathematical Foundations
  - the binomial distribution
  - the normal and lognormal distributions
  - expectation, volatility and Laplace transform
  - the central limit theorem and the law of large numbers
  - time value of money, force of interest, money market account
  - stock models: binomial model and geometric Brownian motion
  - actuarial pricing vs no-arbitrage pricing
  - perfect hedging: forward contract as an example
- Discrete Processes and Binomial Trees
  - one-period binomial model/tree

  - no-arbitrage pricing and risk-neutral probabilities
  - random walk and binomial stock model
  - multi-period recombining binomial tree and the distribution of the risky asset at any given time
  - risk-neutral probability measure
  - pricing options by backwards recursion
  - the CRR formula, pricing options using the binomial distribution
  - Python programming for option pricing
  - American option and valuation
  - self-financing strategy, replicating portfolio
  - complete and incomplete markets
  - construction of binomial models from market information
  - valuation and hedging of variable annuity guarantees with binomial model
  - moving towards continuous models

- Continuous Processes
  - (arithmetic) Brownian motion and properties
  - geometric Brownian motion model for stock prices
  - stochastic integral and properties
  - Ito process and SDEs
  - SDE expression of BM and GBM
  - Ito's lemma and its applications
  - the product rule
  - martingale revisit
  - present value process and martingale
  - Fundamental Theorem of Asset Pricing
  - risk-neutral valuation
  - self-financing trading strategy and replicating portfolio in continuous time
  - application to the Black-Scholes model
  - The Black-Scholes PDE
  - greeks (Delta, Gamma, Rho and Vega) and sensitivity analysis
  - implied volatility
  - simulating stock price paths under the real-world and risk-neutral probability measures, and pricing option by simulation, using Python
  - dynamic hedging variable annuities using greeks

- Pricing Market and Fixed Income Securities

- currency exchange and exchange forwards and options
- options on stocks with dividends
- zero-coupon/discount bonds and coupon bonds
- zero rate, bond yield, short rate and forward rate, and their term structures
- SDE for zero-coupon bonds, the Sharp ratio
- risk-neutral valuation for bonds
- the extended Vasicek/Hull-White short rate model and its Properties
- the affine form of zero-coupon bond prices under Vasicek
- calibration of the extended Vasicek model to zero curve
- simulating short rate paths
- the Cox, Ingersoll and Ross model and its applications
- derivatives on discount and coupon-bearing bonds

Lectures, Office Hours, Assignments and Exams

Lectures

All the lectures will be conducted in person. After each lecture, I will post the pdf of the notes at Quercus. In case that I cannot make to a class I will teach online using the zoom link above. I will use the share screen function to show my notes. If you have a question, use the chat room to post it. I will record that lecture and post it at MyMedia.

Office hours

Thursdays from 1pm to 3pm in person. In addition to the course materials, I am also happy to provide advice on job interview/resume writing, career developments, company information, graduate schools, and things in that nature.

Assignments

I will give 5 homework assignments during the semester and they will be posted at Quercus. Some of the problems involve programing using Python. Email your solutions to act460h1@gmail.com before the due date. One half of the problems in the assignments will be graded. Each assignment is worth 4% toward the final mark. I intend to ask the TA to give a tutorial or two on Python programming basics for financial modelling. Times: TBA.

Assessments

Three one-hour written-answer term tests will be given on Oct 11, Nov 22 and Dec 13 (Dec 13 is tentative) from 2:10 to 3:10pm respectively in class. I will lecture one hour after the tests, starting at 3:30. The first two term tests account for 25% each toward the final mark. The final test account for 30% of the final mark. Should you be forced to miss the midterm, you must contact me to arrange a time within one week for an individual oral makeup test.

The Code of Behaviour on Academic Matters

Visit `www.artsci.utoronto.ca/osai/students`

Relation with the SOA Investment & Financial Markets (IFM) Exam
This course is not designed to prepare for the SOA IFM exam. The topics and mathematical contents covered in this course go much deeper and broader than Topics 6-10 of the IFE. However, the course does help you understand the topics in the exam better.