# ACT460H1F/STA2502 Stochastic Methods for Actuarial Science

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Office hours: Wednesdays 10am-noon, 2pm-4pm or by appointment.

Lecture Times: Tuesdays 2-5pm. Location: RW110.

## Prerequisites: STA347

Having STA347 is a must. If you have not taken ACT370 but did well in STA347 and other math courses, and have some background in finance/financial economics, I will let you take the course. According the FAS regulations, you must submit a waiver form along with your latest transcript to me for approval. The form can be downloaded from

http://www.utstat.toronto.edu/wordpress/wp-content/uploads/2011/09/request-for-prereq-or-coreq-waiver.pdf Please submit a filled waiver form by Thursday Sept 18. If you miss any prerequisites and do not have your waiver request approved, you will be removed from the course on Friday Sept 19.

### Recommended Reference Books

Financial Calculus: An Introduction to Derivative Pricing by Martin Baxter and Andrew Rennie, Cambridge University Press (I strongly encourage you to purchase a copy from Amazon)

Introductory Stochastic Analysis for Finance and Insurance by X. Sheldon Lin, Wiley and Sons

Derivatives Markets by Robert Macdonald

Options, Futures and Other Derivatives by John Hull, Prentice Hall

# **Topics**

I will use Financial Calculus as a main reference and cover the topics in the following order. However, the material to be covered in the course will be substantially more than those in Financial Calculus in terms of breadth and depth.

### • Mathematical Foundations

- The Binomial Distribution
- The Normal and Lognormal Distributions
- Expectation, Volatility and Laplace Transform
- The Central Limit Theorem and the Law of Large Numbers
- Time Value of Money, Force of Interest, Money Market Account
- Stock Models: Binomial Model and Geometric Brownian Motion
- Actuarial Pricing vs No-Arbitrage Pricing
- Perfect Hedging: Forward Contract as an Example

# • Discrete Processes and Binomial Trees

- One Period Binomial Model/Tree
- No-Arbitrage Pricing and Risk-Neutral Probabilities
- Random Walk and Binomial Stock Model
- Risk-Neutral Probability Measure
  - Pricing Options by Backwards Induction
  - Self-Financing Strategy, Perfect Delta Hedging
  - Complete Market
  - Filtration/Information Structure
  - Conditional Expectation with respect to Filtration
  - Martingale
  - Valuation and Hedging of Variable Annuity Guarantees with Binomial Model
  - Moving towards Continuous Models

## • Continuous Processes

- (Arithmetic) Brownian Motion and Properties
- Geometric Brownian Motion Model for Stock Prices
- Ordinary Calculus: Differential and Integral
- Ordinary Calculus: Differential Equations: ODE and PDE
- Stochastic Integral and Properties
- Ito Processes
- Stochastic Calculus: Ito's Lemma
- The Product Rule
- Self-financing Trading Strategy and Replicating Portfolio
- Application to the Black-Scholes Model
- The Black-Scholes PDE
- Greeks and Sensitivity Analysis
- Hedging Variable Annuities using Greeks
- Monte Carlo Methods for Option Pricing
- Nested Simulation and Risk Management of VA Portfolio

# • Pricing Market Securities

- Currency Exchange and Exchange Forwards and Options
- Options on Stocks with Dividends
- Non-tradables and Market Price of Risk
- Quantos

### • Interest Rates

- Zero-Coupon/Discount Bonds
- Yield, Short Rate and Forward Rate
- The Black-Derman-Toy Discrete Time Model
- SDE for Discount Bonds
- Risk-Neutral Valuation
- Short Rate Model
- The Vasicek/Hull-White Model

## Assignments, Test and Exam

I will give 5 homework assignments during the semester. One half of the problems in the assignments will be graded. Each assignment is worth 3% towards to the final mark. There will be 4 tutorials conducted by a TA that will cover EXCEL and VBA programming skills for financial modelling and problem solving skills. A 90-minutes written-answer midterm test will take place on Tuesday October 28 from 2:30 to 4:00. It will account for 34% of the final mark. Should you be forced to miss the test, you are required by faculty regulations to submit, within one week, appropriate documentation from the U of T Health Services to me or to the Departmental Office SS6018 (Print on it your NAME, STUDENT NUMBER, course number, and date.). And you must contact me to arrange a time within one week for an individual oral makeup test. A written-answer final exam (2 hours) will be given during the faculty exam period. The final exam accounts for 51% of the final mark.

### The Code of Behaviour on Academic Matters

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