### **ACT 247: Introductory Life Contingencies (Winter 2023)**

### **Instructor & TA Information:**

Name	Role	Contact
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### **Class times:**

• Lectures: Thursdays 11:10AM to 01:00PM, NL 6.

• **Tutorials:** Tuesdays 10:00AM to 11:00AM, online.

### **High level description**

Probability theory applied to survival and to costs and risks of life assurances, life annuities, and pensions; analysis of survival distributions; international actuarial notation.

#### **Course outcomes:**

By the end of the course, you will be able to:

- Understand the key elements behind the mathematics of pricing key life insurance products: sequences and series, probability theory, and time value at money.
- Recognize life contingency notation and comfortably apply it.
- Understand the concept behind and apply life contingency tables.
- Understand and apply parametric survival models for long-term insurance coverages.
- Understand the key features of life insurance and apply mathematical relationships to price them.
- Understand the key features of life annuities and apply mathematical relationships to price them.

**Textbook:** S. Broverman, Actuarial Science Coursebook for [ACT247H + ACT348H 2022-23] Edition.

### **Course tentative outline:**

- (1) Weeks 1 & 2: Part 1 Foundations (2023-01-08sun  $\rightarrow$  2023-01-21sat):
  - a. Introduction.
  - b. Life insurance products (high level).
  - c. ABCs (Section 1):
    - i. Key geometric series.
    - ii. Key integrals.
    - iii. Probability theory.

- 1. Intro.
- 2. Single variable functions.
- 3. Multiple variable functions.
- 4. Conditional probability.
- 5. Independence.
- 6. Mixture distributions.
- 7. Conditional distributions.
- 8. Distribution functions.
- iv. Time value of money.
- d. Modelling survival & mortality (Sections 2 & 3):
  - i. Survival probability in the context of life and death
  - ii. Time until death.
  - iii. Force of mortality.
    - 1. Of a newborn.
    - 2. Of a person aged x.

### (2) Weeks 3, 4, & 5: Part 2 - Life tables & parametric survival models (2023-01-22sun → 2023-02-11sat):

- a. Life tables (Section 4)
- b. Mean and variance survival metrics (Section 5):
  - i. Complete lifetime expectation,  $T_x$ .
  - ii. *n*-year term expectation of life for x,  $T_{x,n}$ .
  - iii. Median and mode lifetime of  $T_x$ .
  - iv. Curtate expectation of life for x,  $K_x$ .
  - v. *n*-year curtate expectation of life for x,  $K_{x,n}$ .
  - vi. Variance of lifetime.
  - vii. Variance of curtate lifetime.
- c. Parametric survival models (Section 6).
- d. Fractional age assumptions (Section 7).
- e. Select and ultimate mortality (Section 8).
- (3) Week 6: Midterm (2023-02-12sun  $\rightarrow$  2023-02-18sat).
- (4) Week 7: Reading week (2023-02-19sun  $\rightarrow$  2023-02-25sat).

# (5) Weeks 8, 9, & 10: Part 3 - Life insurance products (2023-02-26sun → 2023-03-11sat):

- a. Introduction (Sections 9, 10, 11, 12, & 13):
  - i. Premise.
  - ii. Metrics (single policy).
  - iii. Metrics (multiple policies).
  - iv. Useful recursive expressions (APVs).
  - v. Useful continuous/yearly interchanging formula under UDD assumption.

- b. Constant paying benefits (Sections 9, 10, 11, & 12):
  - i. n-year term insurance.
  - ii. Whole life insurance.
  - iii. n-year pure endowment.
  - iv. *n*-year endowment insurance.
  - v. Special n-year endowment insurance.
  - vi. *n*-year deferred insurance.
  - vii. *n*-year deferred *j* year term insurance.
- c. Varying paying benefits (Sections 11, & 12):
  - i. Geometrically increasing benefit.
  - ii. Whole life increasing insurance.
  - iii. n-year term increasing insurance.
  - iv. *n*-year term decreasing insurance.
  - v. Other continuous varying benefit products.
- d. Simplifications when assuming distributions (Section 13):
  - i. Uniform.
  - ii. Exponential.
  - iii. Normal approximation.
- e. Mathematical relationships between insurance products (Section 13):
  - i. Simple, direct relationships.
  - ii. Recursive relationships.
  - iii. Discrete and continuous insurance relationships assuming UDD.
  - iv. Discrete and fractional (whole life insurance only).
  - v. Covariances.

# (6) Weeks 11 & 12: Part 4 - Annuity insurance products (2023-03-12sun → 2023-03-25sat):

- a. Introduction (Sections 14, 15, 16, & 17).
- b. Discrete life annuities (Section 14 & 15):
  - i. Whole life annuity due.
  - ii. *n*-year temporary life annuity-due.
  - iii. n-year deferred life annuity-due.
  - iv. Simplifications under known probability distributions.
- c. Continuous life annuities (Section 16):
  - i. Continuous whole life annuities.
  - ii. Continuous *n*-year temporary life annuity.
  - iii. Continuous *n*-year deferred whole life annuity.
- d. Mathematical relationships between life annuities (Section 17):
  - i. Recursive relationships.
  - ii. Life annuities with *m*-thly payments.
  - iii. Varying life annuities.
  - iv. Modified mortality risk & structured settlements.

#### (7) Weeks 13: Revision (2023-03-26sun $\rightarrow$ 2023-04-01sat).

### **Course Grading:**

- 1) Assignments (40%):
  - a. Assignment 1 (Part 1): 10% (due 2023-01-27Fri 11:59PM)
  - b. Assignment 2 (Part 2): 10% (due 2023-02-17Fri 11:59PM)
  - c. Assignment 3 (Part 3): 10% (due 2023-03-17Fri 11:59PM)
  - d. Assignment 4 (Part 4): 10% (due 2023-03-31Fri 11:59PM)
- 2) Open book, online, timed MCQ tests (60%):

Standard weights:

- a. Midterm (Parts 1 & 2): 20% (on 2023-02-16Thu 11:00AM to 01:00PM)
- b. Final (Parts 1, 2, 3, & 4): 40% (TBD)
  The 40% weight follows the following formula:  $\max(0.2Midterm + 0.4Final, 0.6Final)$

**Academic integrity:** Three key principles are held in this course: fairness and transparency from the instructor and a solid work ethic from the student. Thus, anyone caught cheating (e.g., copying assignments, solving individual-based assessment problems in groups) will be met with the strictest penalties provided by UT's guidelines.

### Canadian Institute of Actuaries (CIA)'s University Accreditation Program (UAP):

This course is one of the mandatory courses under Canadian Institute of Actuaries (CIA)'s University Accreditation Program (UAP). UAP has moved away from the course-by-course accreditation method and towards program accreditation method (the "Pathway 1 of CIA qualification"). Under the new pathway, in order to obtain ACIA (Associate of CIA) professional credential, students need to:

- 1. Complete a degree from an actuarial program (ACT Specialist or Major) at University of Toronto and pass a list of mandatory courses. No minimum course grade or GPA is required as long as students pass all the mandatory courses. The full list of UofT's 16 mandatory courses are: ACT240, ACT245, ACT247, ACT348, ACT349, ACT370, ACT451, ACT452, ACT466, STA257, STA261, STA302, STA314, ECO101, ECO102, MGT201/RSM219.
- 2. For transition: CIA will accept an actuarial degree from UofT completed between June 30, 2015 and October 31, 2023 without all the specified mandatory courses.
- 2. Complete the ACIA Module (administered by CIA, projected Spring 2023).
- 3. For transition: a student can be exempt from the ACIA Module if they complete SOA exam PA and the 8 FAP Modules and assessments by December 31, 2023.

- 3. Complete an open-book ACIA Capstone Exam (administered by CIA, projected Fall 2023).
- 4. For transition: a student can be exempt from the capstone exam by completing any combination of UAP credits or exams for P, FM, IFM, LTAM, STAM and SRM by October 31, 2023. The deadline to apply for UAP credits is September 30, 2023.

Details on the new pathway for students can be found here: <a href="https://education.cia-ica.ca/acia/acia-for-accredited-university-students/">https://education.cia-ica.ca/acia/acia-for-accredited-university-students/</a>

### **Textbook purchase information:**

You may purchase the coursebook on the UofT Bookstore website.

Navigate to the **Digital Course Materials** section on the University of Toronto Bookstore Website at <a href="https://uoftbookstore.com/textbooks/access\_codes.asp">https://uoftbookstore.com/textbooks/access\_codes.asp</a>?

From here, scroll down the list and select your course, which appears as:

STG ACT 230 Coursebook

STG ACT 240/245 Coursebook

STG 247/348 Coursebook

STG 348/455 Coursebook

Follow the instructions to **purchase and download your code**. You will need to check out via the **eBook Shopping Cart** on the page to purchase digital materials, not the regular cart. See screenshot below.

