# ACT460/STA2502 Stochastic Methods for Actuarial Science

## Fall 2021

Day/Time: Tue 2pm-5pm (Online)

Platform: Quercus and MS Teams

Instructor: Yuchong Zhang

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**Course website:** Quercus (Course materials provided on Quercus are for the use of students currently enrolled in this course only. Sharing course materials with anyone outside of the course is unauthorized use.)

**Description of this course:** Mathematical theory and probabilistic tools for modeling and analyzing security markets are developed. Topics include pricing derivative securities in complete and incomplete markets, Brownian motion and stochastic calculus, the Black-Scholes model, and term structure of interest rates. We will spend about one third of the course on discrete-time models, and two thirds of the course on continuous-time models. Prerequisite to this course are ACT240, ACT 245 and ACT247 with grades 63% or above. In addition, you should be comfortable with probability theory at the level of ACT350/STA347. ACT370 is also recommended.

#### **Reference Books:**

- STEVEN SHREVE: Stochastic Calculus for Finance I & II. (Main reference) (Please check http://www.math.cmu.edu/users/shreve/ for errata.)
- TOMAS BJÖRK: Arbitrage Theory in Continuous Time.
- MARTIN BAXTER AND ANDREW RENNIE: Financial Calculus: An Introduction to Derivative Pricing.
- X. SHELDON LIN: Introductory Stochastic Analysis for Finance and Insurance.
- JOHN HULL: Options, Futures and Other Derivatives

**Grading:** Your course grade will be determined by the performance on two term tests  $(2 \times 30\%)$  and a final exam (40%).

#### Exam dates and policy:

• Term tests (in-person): October 12, 2021 and November 23, 2021 at Sidney Smith 2106. Should you be forced to miss the test, you are required by faculty regulations to submit, within one week, appropriate documentation, and you must contact me to arrange a time within one week for an individual oral makeup test.

• Final exam (in-person): TBA. All students must take the final at the time scheduled by the university.

**Problem sets:** There will be 4-5 ungraded problem sets for you to practice your understanding of the course material. While you may discuss these problems with a partner, it is strongly recommended that you first attempt them by yourself.

Tutorials: There will be two TA-led tutorials to cover basic Python skills for financial modeling.

### Academic Integrity:

The University of Toronto's intellectual community relies on academic integrity and responsibility as the cornerstone of its work. As a student, you alone are responsible for ensuring the integrity of your work and for understanding what constitutes an academic offence. Please visit http://www.artsci.utoronto.ca/osai/students for the rules and expectations, and tips on how to avoid committing an academic offence. Failure to observe these rules of conduct will have serious academic consequences, up to and including expulsion from the university.

The following is a tentative guide as to how the course will proceed:

- Lecture 1: Derivative security, no-arbitrage pricing
- Lecture 2: Binomial model, review of probability theory, martingales in discrete time
- Lecture 3: Risk-neutral pricing, CRR formula, binomial model calibration
- Lecture 4: American options, trinomial model, incomplete markets, from random walk to Brownian motion
- Lecture 5: Term Test 1, Tutorial 1
- Lecture 6: Continuous time stochastic processes, Markov processes and martingales in continuous time
- Lecture 7: Reflection principle, barrier options, quadratic variation, arithmatic and geometric Brownian motions
- Lecture 8: Stochastic integral, Itô's lemma, stochastic differential equations
- Lecture 9: Black-Scholes analysis, Greeks, implied volatility
- Lecture 10: Term Test 2, Tutorial 2
- Lecture 11: Risk-neutral measure, Fundamental Theorem of Asset Pricing, risk-neutral pricing
- Lecture 12: Short rate models